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Graded Qualities

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Abstract

The idea that qualities can be had partly or to an intermediate degree is controversial among contemporary metaphysicians, but also has a considerable pedigree among philosophers and scientists. In this paper, we first aim to show that metaphysical sense can be made of this idea by proposing a partial taxonomy of metaphysical accounts of graded qualities, focusing on three particular approaches: one which explicates having a quality to a degree in terms of having a property with an in-built degree, another based on the idea that instantiation admits of degrees, and a third which derives the degree to which a quality is had from the aspects of multi-dimensional properties. Our second aim is to demonstrate that the choice between these account can make a substantial metaphysical difference. To make this point, we rely on two case studies (involving quantum observables and values) in which we apply the accounts in order to model apparent cases of metaphysical gradedness.

> Perjury, perjury, in the highest degree;
> Murder, stern murder, in the direst degree;
> All several sins, all used in each degree
>
> Shakespeare
> *The Tragedy of Richard the Third*

1 Introduction

Current metaphysical orthodoxy has it that things either have or lack qualities, *tertium non datur*. No matter whether they rely on universals, properties, tropes, sets, or mereological fusions to account for the qualitative nature of things, all orthodox accounts imply that it is an all-or-nothing matter whether

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an object has or lacks a quality.[1] This seems fine in some cases. Objects either have mass, or don't; natural numbers are either prime, or not; the performance is sold out or it isn't. But there are also qualities which do not fit the orthodox view as nicely. Whether a tool is useful, a person is healthy, a business is lucrative, or a piece of information important, all clearly are matters of degree.

In this paper, we assume that the apparent gradedness in having certain qualities is a phenomenon in need of a metaphysical explanation. A significant number of authors, both contemporary and historical, have either entertained, or even accepted a view of this kind. However, this approach is bound to be controversial. As Ney, who is herself sympathetic to the idea that certain properties can be instantiated to a degree, points out:

> [t]raditionally in analytic philosophy, we are not used to thinking of properties as instantiated only to a degree. Rather we tend to think of a property's being instantiated as an all-or-nothing matter. Entity e is either F or not F. It cannot be F only to a degree. To reject this would be to reject classical logic. (Ney (2020), p. 4245)

For some philosophers, the thought that there are graded qualities is not just unusual, but outright incoherent. Armstrong calls the idea difficult (Armstrong (1989), p. 79) and even remarks that we cannot 'make much sense of degrees of instantiation.' (Armstrong (1989), p. 48.) Russell categorically rejects the idea, saying that '[n]othing is more or less what it is, or to a certain extent possessed of the properties which it possesses.' (Russell (1923), p. 85.)[2] In a somewhat similar vein, Lewis *de facto* excludes graded properties from his ontology by committing to 'a treatment of properties that requires things to have or to lack them simpliciter.'[3]

Our first objective is to argue that one can, *pace* Armstrong, make sense of the idea that there are graded qualities. We do this by formulating a set of minimal conditions which theories of such qualities have to meet and then introducing three such theories. This also allows us to address the worry expressed by Ney: While at least one of the three theories can be fleshed out in a

[1] Throughout the paper, we use 'quality' as an ontologically neutral term to talk about what entities are like. Particular ontological notions like property, trope, set, or fusion are used in the context of the metaphysical accounts of the targeted graded quality which we will present in this paper.

[2] Note that Russell equates having a quality to an intermediate degree with it being indeterminate whether the relevant object has the quality. We take this to be a substantial assumption in need of further argument; the three theories of metaphysical gradedness introduced in §3 are neutral regarding this point.

[3] Lewis (1986), p. 53. Note that in the quoted remark, Lewis targets relative instantiations of properties, not grades of instantiation. He does mention 'properties which admit of degrees' immediately after the quoted remark, but it is clear from the context that what he means by this are properties which have an 'internal degree structure', like mass, not properties which can be instantiated to an intermediate degree. This does of course not undermine the point that his quoted claim rules out that there are properties which are instantiated to a degree.

nominalistically acceptable way using a non-classical logic, this is only an option and not strictly mandated by the theory.

In a second step, we discuss two case studies which a) show that theories of graded qualities can play particular theoretical roles in metaphysics and b) demonstrate that the choice between the three theories can make a substantial difference. One conclusion we can draw based on the case studies is that the three accounts differ in how well they are adapted to the different kinds of qualities we focus on. We take this to suggest a pluralist view about metaphysical gradedness.

We should further clarify our target notion by distinguishing gradedness in our sense from three other notions of gradedness familiar from the philosophical literature. First, 'grade' in the metaphysical literature on quantities is used to denote the amount to which an object has a quantity, such as e.g. velocity, height, or duration. (See e.g. Eddon (2013) for a recent overview.) Second, metaphysicians often use 'grade' to talk about positions in an order imposed by a relation, such as e.g. that of relative fundamentality. (See e.g. Correia (2021).) To bring out one important difference between these two uses and what we mean by 'grade': if two objects have the same graded quality to different degrees, they differ in how close they are to fully having the quality. In contrast, it makes little sense to claim e.g., that an object which weighs 4 kg is closer to fully having the mass-quality than an object which only weighs 2 kg.[4], A further important difference to quantities is that, at least prima facie, quantities intrinsically characterize the things which have them in terms of *how much* of them they have (cf. Wolff (2020), p. 2). The grade to which a graded quality is had does not make a similar contribution to intrinsically characterizing the object.[5] Third, following the lead of the literature in linguistics on graded adjectives (see e.g. Schwarzschild (2008)), philosophers of language, but also epistemologists and ethicists use the notion to denote numerical values used to spell out truth-conditions for comparatives or superlatives. The main difference between this use and ours is that the former is metaphysically neutral, in the

[4]This is not to say that the view that graded qualities are, or are at least explainable in terms of quantities is indefensible. One of the views which we will discuss, the degreed properties view, could be developed along such lines.

[5]Note that the degrees to which graded qualities can be had have upper bounds, which might tempt one to think that that they belong to the same category as quantities with an upper bound such as velocity, i.e. that they are really just specific quantities. But this would be mistaken, since having an upper bound is only necessary, not sufficient for being a graded quality, as the difference in intrinsic nature just noted and the seemingly brute fact that graded qualities like wisdom are clearly not quantities underlines. One should also distinguish between an upper bound in the value a quantity can attain and the maximal degree to which a graded quality can be had. The former can change between different qualities, the latter not, since it is not determined e.g. via a specific natural constant, but is rather essentially defined as being the absolute maximum of the degrees to which a quality can be had. We thank an anonymous reviewer for this journal for prompting us to address this point.

sense that it is insensitive to exactly the sort of differences we are interested in, including the ones just highlighted.

2 Motivation

Why take the idea that there are graded qualities metaphysically seriously? There are both historical, as well as contemporary philosophical claims which appear to postulate graded qualities, motivating the search for a way to better understand them.

Let us begin with an important historical view, Plato's theory of Forms. A central motive in Plato's metaphysics is that certain qualities like beauty, justice, and goodness correspond to Forms. Forms are abstract entities which explain why observable things have the corresponding qualities, but are themselves separate from the observable things which have them.[6] That having a Form is a matter of degrees is evident in two central aspect of Plato's view, namely that sensible objects never have the quality corresponding to a Form to the highest degree, and that the only entity which has the quality to the highest degree is the Form itself.[7]

The henological argument for the existence of God involves a further historical view which explicitly assumes the existence of graded qualities. The version from Thomas Aquinas' *Summa Theologiæ* for example contains the following passage:

> The fourth way is based on the *gradation observed in things*. Some things are found to be more good, more true, more noble, and so on, and other things less. But such comparative terms describe varying degrees of approximation to a superlative; for example, things are hotter and hotter the nearer they approach what is hottest. (Aquinas (1964–80), Ia, 2, 3).[8]

In the late 19th century, Bradley similarly proposed a theory of degrees of reality and of truth, according to which 'to be more or less true, and to be more or less real, is to be separated by an interval, smaller or greater, from all-inclusiveness or self-consistency.' (Bradley (1893), p. 364.)

In a more contemporary context, van Woudenberg and Peels (2018) have argued that there are certain degree-involving sentences (their examples be-

[6] These are the two principles (Causality) and (Separation) formulated in Rickless (2020), sec. 3.

[7] See Meinwald (1991), p. 153.

[8] We should note that an application of the multidimensional properties account (see §3.5) in this context leads to an inconsistency with the doctrine of the identity of the divine attributes (cf. Vallicella (2019), 3.3) which Aquinas accepts. We thank Damiano Costa for pointing this out to us.

ing 'Socrates is wiser than Diogenes.', 'Jane is more intelligent than Jack.', and 'Achilles runs faster than the tortoise.'), which refer to particular complex properties that are sensitive to a degree. Their paper seems to offer the only contemporary discussion of gradedness which focuses on metaphysical questions.[9]

Three possible motivations have recently been proposed in the philosophy of physics. Ney has argued that wave-function realism, the view that only the wave function is fundamental and that the existence of all other physical objects is ultimately 'determined' by it, suggests an ontology of graded properties:

> In the general case in which the wave function has nonzero amplitude at multiple locations in its space, it is helpful first to think of the configuration of the particles as a property that may be attributed to them. Then, a natural thing to say is that multiple configurations of the N particles will be instantiated by the wave function, *each to a degree* equal to the amplitude of the wave function at that point squared (Ney (2020), p. 4245, italics added).

Myrvold has embraced a view on which gradedness is even more pervasive:

> *All of the dynamical properties* that make a body a body *are a matter of degree*, and it would not detract at all from their status as bodies if the intensity of these properties did not go abruptly to zero. (Myrvold (2017), p.107, italics added.)

Relatedly, some philosophers have recently proposed that a case for graded qualities can be made in the context of non-relativistic quantum mechanics, when, assuming some (admittedly controversial) interpretations, the properties of a quantum system may (in a metaphysical sense) lack a determinate value. In this context, Calosi and Wilson (2019) suggest that a possible way of understanding such quantum indeterminacy is by means of graded qualities. We will discuss this proposal in further detail in §5.1.

Also in contemporary debates in metaphysics (extended simples occuyping a region to a degree – Simons (2004), p. 377), the philosophy of language (presupposition of graded properties in the fuzzy theory of vagueness – Smith and Rosen (2004), p. 186), of logic (truth as a gradable property – 2021, Section 5), of mind (grades of consciousness – Lee (2023), footnote 2), and in aesthetics and ethics (gradability of axiological qualities – Scruton (2009), p. 9), philosophers argue – or sometimes they just consider it simply a *fact* – that some qualities admit of degrees.

[9] We come back to their account in §3.5, footnote 17.

We are *not* claiming that one can *only* make sense of some (if not all) of the examples above by adopting a metaphysically substantial view of what gradedness amounts to. What we do claim is that doing so is both feasible and comes with a benefit in *systematicity*.

An account of graded qualities in particular provides us with a metaphysical explanation of salient 'partly talk'. Consider the following quote by Dirac: 'As long as the photon is partly in one beam and partly in the other, interference can occur when the two beams are superposed' (Dirac, 1930, pp. 8–9). Or Simon's claim that when we ask 'What is there?' about a subregion of an extended region which is wholly occupied by a basic particle P, the correct answer sometimes is 'P, and P is partly there and partly elsewhere'.(Simons (2004), p. 377.) A straightforward, and a priori plausible way to understand both takes them to indicate that the relevant particle is at one location at a time to a degree, and at another location to (the same or) another degree, where location is a graded quality.

As Simons points out (ibid.), the 'partly'-claims we are interested in should not be confused with claims about the qualities of (proper) parts of objects. In the given examples, this is clear since basic particles do not have parts, at least not as far as our best physical theories tell us. Contrast this sort of 'partly'-claim with, say, 'The chessboard is partly white,' where 'partly' indicates that some parts of the board *fully* (i.e. not just partly) have the quality of being white. Only 'partly'-claims of the former sort are interestingly connected to the ontology of graded qualities, so we focus solely on them.[10]

There is a further, different use of 'partly' which is relevant to our inquiry. Consider the claim that Alcibiades is partly wise. Wisdom is a complex quality which consists (in a way to be further spelled out) of different aspects of being wise. That Alcibiades is partly wise might for example mean that he exhibits *some* of these aspects, perhaps that of being good at calculations or geometry or that of being earnest, but *not others*, e.g. possessing a rich and diverse body of knowledge.

'Partly'-talk is also prevalent in everyday language. Think of, e.g. the claim that a student of pottery partly mastered this craft, or that an operation was partly successful. Accounts of graded qualities provide us with a tool to make sense of such claims. In the rest of the paper we flesh this out in a bit more detail. In particular, we show how to recover 'partly talk' within each of the three metaphysical accounts we discuss. Similarly, an account of graded qualities may provide us with a metaphysical backing for a semantic account of predicate modifiers like 'somewhat' or '-ish': The truth of 'Sam is somewhat

[10] 'Partly'-claims if read as claims about distributional properties, like e.g. *being polka-dotted* (cf. Parsons (2004)), fall outside the scope of our investigation for similar reasons.

wise' may (partly) be explained in terms of Sam's having the quality of being wise to a a certain degree, but, to use technical terms we will introduce in §4, still falling short of either having it *simpliciter* or *fully*. The same holds for 'Sam is wise-ish'.

We take it to be fruitful to attempt to explain an unfamiliar notion in other, more familiar terms. All three accounts of graded quality which make up our following partial taxonomy are '(mildly) reductive', in the sense that they explicate what gradedness consists in using distinct, more familiar metaphysical notions and categories.

3 A taxonomy of metaphysical accounts of graded qualities

In this section, we present a partial taxonomy of metaphysical accounts of graded qualities. The taxonomy is only partial, because we focus on three such accounts, even though several others could be developed.[11] We believe that the loss in comprehensiveness is somewhat offset by a gain in depth: the partial taxonomy is already rich enough to illustrate that the particular way in which one conceives of graded qualities makes a substantial difference in the context of certain metaphysical claims and theories (see §5). Furthermore, we have something general to offer for further work on the taxonomy, namely a set of minimal conditions which, we submit, any account of graded qualities has to meet.

We should also point out that while we mostly use realist language about qualities and properties, we later suggest nominalistically acceptable formulations of at least one of the account we present.

3.1 Some minimal constraints on accounts of graded qualities

There are certain adequacy conditions any metaphysical theory of graded qualities should meet. As such conditions tend to be, they are in a sense unsurprising. Yet, making them explicit is important, since they provide a common basis for the discussion and prevent some potential misunderstandings.

Intermediate Degrees: Graded qualities can be had to different degrees at different times, where some of these degrees are intermediate.

[11] One idea for a further account suggested to us by Kit Fine introduces sentential degree operators of the form '... obtains to degree d' and recovers graded properties in terms of lambda-abstracts of sentences involving such operators.

The point of this condition is to ensure that the degrees to which a graded quality can be had are not exhausted by the binary distinction between fully having and fully lacking it, which would trivialize graded qualities. In general, it seems a sensible assumption that whenever an object has a graded quality at a time, it has it to a degree $0 < d \leq 1$, where d usually ranges over the real numbers. For the first two theories we will introduce, this can be the full range of real numbers, in case of the third, sometimes only a restricted range of 'degree values'. To explain the use of '$<$' instead of '$\leq$': it is unproblematic to equate '*fully* having a quality $\mathcal{Q}$' with 'having the quality $\mathcal{Q}$ to $d = 1$', but, at least for some possible accounts, including the first of our three, equating '*fully* lacking a quality $\mathcal{Q}$' to 'having the quality $\mathcal{Q}$ to $d = 0$' is problematic.

Absolute Qualities: There might be *absolute qualities*, qualities which *cannot* be had to an intermediate degree.

There plausibly are qualities which an object either fully has or fully lacks (think of being greater than 1). The idea underlying the second condition is that an adequate account of graded qualities should not force us to posit gradedness where there is none. Accordingly, it makes sense to clearly distinguish between the two notions of *absolutely* and *fully* having a quality $\mathcal{Q}$. The former notion applies in case a relevant particular has $\mathcal{Q}$ and $\mathcal{Q}$ is an absolute, non-graded quality. The latter in case the relevant particular has a graded quality $\mathcal{Q}$ to degree $d = 1$, i.e. to the maximal degree.

Degree-Connections: There are systematic connections both among graded qualities and between graded qualities and absolute qualities which constrain under which condition and to which degree objects may have them.

To give an example of a degree-connection between two graded qualities, consider the following systematic connection between wisdom and foolishness: Being wise to some degree rules out being *fully* foolish, that is, foolish to the maximal degree. For an example of a degree-connection between a graded and an absolute quality, consider again wisdom and the the quality of knowing that 1 plus 1 equals 2. If someone knows that 1 plus 1 equals 2, where knowing this is an absolute, rather than a matter of degree, then that person is wise to some (however minuscule) degree. An adequate theory of graded qualities has to be able to account for such connections.

Comparative Relations: Objects which have the same graded quality may stand in comparative relations to each other.

Degrees are ordered and ordered things are comparable (think of e.g. Socrates's being wiser than Callicles), so any salient account of graded properties has to

make metaphysical sense of comparative relations in case they obtain. There are several interesting and important questions about the underlying orders, their origin, and nature, which for the most part, we cannot discuss in any detail here. Strictly speaking, the accounts we present here are hence more akin to incomplete templates, than to fully spelled out metaphysical accounts. This in particular concerns our suggestions for how these accounts can meet the last two constraints. To meet them, the degrees postulated by an account of graded qualitues obviously need to be ordered. A fully fleshed-out account can reasonably be expected to not simply *assume* such orderings, as we will admittedly do in the following, but to *justify* and metaphysically explain them. Only this way, the account can ultimately guarantee that the numerical degrees it relies on are more than empty labels.

To partly defend our assumption, we want to point to out that at least in some cases, the particularities of the relevant quality all but guarantee that the degrees to which it can be had are suitably ordered. In case of the observables discussed in §5.1, degrees of having for example map to probabilities, providing us with a rather solid justification for the assumption that the order of the degrees of having parallels their numerical structure.

As for another illustration, think again of the so-called *henological argument* (see the quote from Aquinas in §2 or Anselm's *Monologion*), according to which beings are ordered into levels of perfection which culminate in God. One may in this case take *perfectness* to be a graded quality, where the degree to which it is had by an object tracks an objective distance to the most perfect being.

We should finally mention a notable imbalance between the first two and the third account presented in the next subsections with respect to the issue of a metaphysical explanation of the order of degrees of having. The intrinsic structure of the properties posited by the third account provide it with a resource which one can naturally rely on to explain this order. In comparison, the first and the second account are less complete, since they do not posit comparable metaphysical resources. To show that the numbers which these two accounts attach to the members of families of degreed properties and to degrees of instantiation respectively are not merely meaningless labels, further work will have to be done. As we have pointed out, relevant applications and examples might provide significant insight for such work.

Partiality Graded qualities can be had partly.

Whenever something has a graded property to an intermediate degree, this licenses claims about it partly having this property, in the particular sense specified in the previous section. E.g. someone who is virtuous to an intermediate degree can be rightly said to be partly virtuous. This last condition marks

an important difference between theories of graded properties and theories of other kinds of complex properties which involve a degree structure, such as e.g. quantities, which do not support such 'partly'-claims.

3.2 Degreed properties

How can an object have a quality to a degree? One way to begin answering this question is to focus on the apparent structure of a state of affairs in which an object (allegedly) has a quality to a degree. Such a state of affairs may be thought to involve an instantiation of the three-place relation 'x has $\mathcal{Q}$ to degree d' by an object, a quality, and a degree. The first account rejects this picture in favour of one in which the state of affairs involves not a three-place relation, but a property which itself involves, in a sense to be specified, the degree and applies *absolutely* to an object.[12] Call such properties *degreed properties*. The two ideas introduced here are captured by the following two principles:

Complex Properties : Graded properties contain the degree to which they are had.

Binary Application : Graded properties are had absolutely, that is, either apply or do not apply to the objects which have them (to a degree), where application/non-application is a binary matter.

Let us look at these two principles in turn.

3.2.1 Complex Properties

According to **Complex Properties**, degreed properties involve, or contain a degree. Their internal structure involves two elements: a base property, so to speak, and a degree. Consider the quality of being wise as an example. According to the suggestion we are exploring, this quality does not correspond to a single absolute property; rather what one has in one's ontology are a number of complex, or structured properties such as 'being wise$_{d_i}$', where d_i is the relevant degree.

In effect, let us make the following assumption for now: for every real number in the interval $(0, 1]$ we have a corresponding structured degreed-property P_{d_i}, i.e. a property of 'being P to degree d_i'.[13] With this assumption in place we get the following picture: for each graded quality we really have an uncountable set of degreed properties $S_P = \{P_{d_i | i \in (0,1] \subset \mathbb{R}}\} = \{..., P_{d_{\frac{1}{2}}}, ..., P_{d_{\frac{2}{3}}}, ..., P_{d_1}\}$ involving

[12] (Machina, 1976, p. 58) comes at least close to endorsing an account of this kind in his classic development of the degree-theory of vagueness.

[13] How to account for the internal structure of degreed-properties is a substantial question, which we cannot discuss here.

the base property P which corresponds to $\mathcal{Q}$. In line with the general assumption about *fully* lacking a property mentioned in §3.1, the interval we use for the degrees is only half-closed. Taking a closed interval would result, in the case at hand, in the existence of a property involving the degree $d = 0$. This is arguably extremely problematic—making fully lacking a quality a matter of having the corresponding zero-degree degreed property—hence our initial assumption. It should also be clear that the $<$-relation on $(0, 1] \subset \mathbb{R}$ can be used to define an *order* on S_P which we later use to show how the account meets Comparative Relations.

3.2.2 Binary Application

The second principle tells us that degreed-properties are either had or not had by an object, where this disjunction is exclusive and where 'having a quality' is hence a binary matter. This aspect of the degreed-property view is perfectly conservative with respect to standard theories of qualities—which do not allow them to be had to a degree. Having a property remains an absolute matter, even in the presence of gradedness: you either have a degreed property or you don't. This of course means that the account is perfectly compatible with classical logic.

But it also entails that, from a metaphysical perspective, 'having to a degree'-talk is not to be understood literally. According to the degreed properties account, no object has a property *to a degree*. Rather, having a quality to a degree is explained in terms of *absolutely having a degreed-property*.

Taken together, the two principles illustrate that the degreed properties view is metaphysically conservative in one (incorporates an orthodox view of instantiation), metaphysically revisionary in another sense (posits more properties than one might think there are).

3.2.3 Degreed properties provide an account of graded qualities

It remains to be verified whether the account satisfies the minimal requirements we laid down in §3.1. The account clearly satisfies the first, namely Intermediate Degrees, since it allows things to have a quality to different degrees (by having different corresponding degreed properties), including to intermediate degrees (by having a degreed property P_{d_i} with $i \neq 1$).

Consider Absolute Qualities next: The account says that for every graded quality, having it means (absolutely) having one of an (uncountable) set of degreed-properties, not that this is the case for every quality.

What about Degree-Connections? Reconsider our example of wisdom and foolishness and the idea that someone who is wise to some degree cannot

be maximally foolish. To account for this degree-connection, one can impose the following constraint: $\forall x(\exists d(W_d(x)) \rightarrow \forall d'(F_{d'}(x) \rightarrow d' \neq 1))$.

Is Comparative Relations also satisfied? Assuming that the set S_P can be ordered using $<$, the degreed properties account straightforwardly meets the requirement: if it is the case that x is $\mathcal{Q}$-*er*, or more $\mathcal{Q}$ than y, this is so simply because there is a familiy of degreed properties S_P, such that x has $P_{d_i} \in S_P$, y has $P_{d_j} \in S_P$ and $i < j$ and the base property P of that family corresponds to $\mathcal{Q}$. By 'corresponds', we mean that the property is used within the theory, possibly together with other entities, to account for the quality.

Finally, we have to address whether degreed properties satisfy Partiality. The following schemata do just that:

From Degreed Properties to Degrees: x has a quality $\mathcal{Q}$ to degree d_i if, and only if, x is P_{d_i}, where $P_{d_i} \in S_P$ and each element of this set involves the property P which corresponds to $\mathcal{Q}$.

Degreed Properties and Partly-Talk: x *partly* has a quality $\mathcal{Q}$ if, and only if x has $\mathcal{Q}$ to degree $d_i < 1$.

3.3 Degreed instantiation

One might argue that the degreed properties account, metaphorically speaking, puts the degrees in the wrong place: when an object has a quality to a degree, then given an ontology of properties, what comes in degrees is the relation of having, not the property. The second account spells out this idea.

For the sake of illustration, consider again a state of affairs involving the schematic three-place relation 'x has $\mathcal{Q}$ to a degree d'. If the degree is supposed to modify the relation of having, what would this state of affairs have to look like? According to the second account, the answer is simple: besides the object and the relevant property, the state of affairs contains a relation of *instantiation*—or having—which ties the object and property together, and it is this relation which comes in degrees.[14] The three core ideas of this second account are:

Simple Properties : Properties are simple and do not involve the degrees to which they are had.

Mediacy : Properties apply to objects by standing in the instantiation relation to them.

Graded Instantiation : The instantiation relation admits of degrees.

[14]This second account is analogue to a 'Platonic' theory of quantities, a kind of theory which Bigelow and Pargetter (1988) briefly entertain and then dismiss.

We again discuss these principles in turn.

3.3.1 Simple Properties

This principle is straight-forward; it just says that the account does not admit any sort of unorthodox qualitative properties to make sense of graded qualities.

3.3.2 Mediacy

According to **Mediacy**, for an object to have a quality is for it to stand in the relation of instantiation to a property. One might worry that this seeming reification of properties makes the account unacceptable by nominalist standards. We address this worry below, providing two ways of understanding the instantiation-relation which are compatible with (some varieties of) nominalism.

3.3.3 Graded Instantiation

Graded Instantiation is the characteristic principle of the second account, since it fixes the place degrees take in its ontology: the degree to which a quality is had by an object consists in the degree to which the object instantiates a corresponding property, i.e. it attaches to the relation of instantiation which ties object and property together. But what does it mean to say that the degree attaches to this relation?

Metaphorically speaking, the relation ties an object and a property together with a certain strength; the closer the degree of instantiation is to the maximal degree 1, the closer it is to fully instantiating the property. This notion of 'fullness' indicates the 'maximal strength' with which an object can instantiate a property.

There seem to be at least two ways to understand graded instantiation. According to the first, the degree is built into the relation of instantiation and forms with it an inseparable whole. This seems to make instantiation a degreed relation, i.e. a relation of the same kind as the properties posited by the previous account. This first proposal would hence make the second account a somewhat unusual special case of the first which only admits one degreed relation and uses it to account for graded qualities. Here, we will instead propose a second way, which treats 'instantiation' as a *sui-generis* relation.

The disadvantage of treating instantiation as sui generis is that this makes a general objection to any theory which works with primitive notions salient: if graded instantiation is neither reducible to, nor reductively explainable in terms of other notions, the notion is obscure, or even unintelligible. As Bigelow and Pargetter, who discuss an analogue schema for theories of quantities, put it: 'the notion of 'degrees of a relationship' cries out for analysis.' (Bigelow and

Pargetter (1988), 289-90) Unlike them, we do not think that this is a reason to discard the second account: the intelligibility objection can be met. Interestingly enough, the answer we propose reveals at the same time the compatibility of the account with (two forms of) nominalism, thus taking care of the realist worry mentioned in the brief discussion of Mediacy.

Nominalists commonly rely on two ways to model property instantiation. The first models properties as sets and instantiation as elementhood. Accordingly, a property is represented by a set and an object instantiates the property just in case it is an element of that set.

The second way models properties as fusions of objects and instantiation as parthood. More specifically, any property is represented by a mereological fusion of objects and that an object instantiates a property is modelled by that object's being a part of the fusion. Both of these proposals can be generalised to model degrees of instantiation by building a degree into the formal relation which is used to model instantiation.

In case of the set-based approach, instantiation by degree can be modelled by fuzzy sets. Fuzzy set-theory provides a well-understood formal framework which characterises fuzzy sets and gradual membership in them. (Cf. Zadeh (1965).) In case of the mereological approach, fuzzy mereology takes the place of fuzzy set-theory. (Cf. Cotnoir and Varzi (2021), §6.3.3.)

So one can indeed make (formal) sense of degrees of a primitive graded instantiation relation. The two nominalistic approaches just mentioned provide us with precise formal accounts of how graded instantiation behaves, which at least to the degree to which the availability of a formally precise model is apt to do so, also addresses the intelligibility-objection. The two approaches furthermore also provide at least the starting point for two distinct answers to the realist worry, for they are basically variations of *class nominalism* and *mereological nominalism*. We will have to leave open whether they can be developed into full-fledged nominalist theories of (talk of apparent) graded qualities of the same explanatory power as realist alternatives.

Having made a case that the degreed instantiation view neither collapses into the degreed properties view, nor falls prey to the intelligibility-objection, we can now turn to the question of whether it yields an adequate account of graded qualities.

3.3.4 Degreed instantiation provides an account of graded qualities

Let us again consider the minimal conditions. As in case of the first account, there is little doubt that the second account satisfies the first condition, Intermediate Degrees by allowing things to instantiate the same property to

different, including intermediate, degrees.

Absolute Qualities is arguably the most interesting requirement in the context of the second account. One way to meet it is to postulate the existence—using realist language—of two instantiation-relations, degreed-instantiation and absolute-instantiation. Equipped with both relations, one can easily distinguish between *fully* (standing in degreed-instantiation to the corresponding property to the maximal degree) and *absolutely* (standing in absolute-instantiation to the property) having a quality. That said, postulating two different instantiation relations seems problematic for various reasons, not the least since it introduces a second piece of primitive ideology.

Fortunately, this is not the only way to meet the requirement. One can instead insist that, while objects always stand in the graded instantiation-relation to properties, some properties, by their very nature, are such that they can only stand in this relation to any object to the maximal degree $d = 1$ or not at all. The distinction between *fully* and *absolutely* having a quality can then be spelled out in terms of this constraint. In both cases, the relevant object has the corresponding property P to degree 1, but it *fully has* P only if it could have had P to a (strictly) lesser degree, whereas it *absolutely has* P, if P's nature rules this out.

What about Degree Connections? As in case of degreed properties, the degree-connection that being wise to some degree prevents one from being fully foolish can again be enforced by imposing a constraint, namely: $\forall x(\exists d(dW(x)) \rightarrow \forall d'(d'F(x) \rightarrow d' \neq 1))$, where the degree-variables d, d' act as predicate-modifiers on predicates standing for simple, that is non-degreed, properties.

Comparative Relations is—once again—easy to meet. In the context of the degreed instantiation account, whenever it is correct to say that x is $\mathcal{Q}$-*er*, or more $\mathcal{Q}$, than y, this is so because x stands in the instantiation-relation to P to degree d_i, y stands in the instantiation-relation to P to degree d_j and $d_i > d_j$, where P corresponds to $\mathcal{Q}$.

Finally, Partiality requires that the graded instantiation can also account for 'partly talk'. The following shows it can:

From Degreed-Instantiation to Degrees: x has a quality $\mathcal{Q}$ to degree d_i iff x instantiates P to d_i, where P corresponds to $\mathcal{Q}$.

Degreed-Instantiation and Partly-Talk: x *partly* has a quality $\mathcal{Q}$ iff x has $\mathcal{Q}$ to degree $d_i < 1$.

3.4 Degrees and Regions

Before we move on to the third account of graded qualities, we would like to note and discuss a structural analogy between 'having a quality to a certain degree d' and 'having a property at a certain region r'. To bring out the analogy, we focus on a specific problem, namely the problem of qualitative heterogeneity. Say that an extended simple is a material object that is spatially extended but does not have any proper parts. Suppose furthermore that one such extended simple x displays some form of *qualitative variegation*. For example, x is half-black and half-white. In this case, under the assumption that x is genuinely simple, qualitative variegation cannot be accounted for in terms of the compositional structure of x, for instance by saying that it has a proper part y_1 which is black, and another proper part y_2 which is white. Note however that we would still like to say that x is *partly* black, and *partly* white.

Two familiar solutions, one relying on what Schaffer calls *regionalised properties*, the other on *regionalised instantiation* (cf. Schaffer (2010), p. 60), closely resemble the two accounts of graded qualities which we have discussed so far. According to the first, seemingly simple monadic qualities such as 'being black' or 'being white' correspond to complex properties such as 'being black$_{r_1}$' or 'being white$_{r_2}$', where r_1 and r_2 are distinct regions of space.[15] In this case, x is partly black and partly white by instantiating (absolutely) the complex properties of 'being black$_{r_1}$' and 'being white$_{r_2}$'. According to the second solution, instead of regionalizing properties—thus making them complex—one regionalises the instantiation of simple properties. Accordingly, 'being black' and 'being white' correspond to simple properties, but they can be instantiated-*at regions*. In particular x is partly black and partly white by instantiating-at-r_1 the simple property 'being black', and by instantiating-at-$_{r_2}$ the simple property of 'being white'.

These two accounts, we submit, are the *regional counterparts* of the degreed-properties and degreed-instantiation accounts. This gives us a further, conditional argument for the intelligibility of graded instantiation, and also for graded qualities in general: Insofar as the analogy holds and insofar as region-sensitive properties are intelligible, graded qualities are intelligible, too.

3.5 Multidimensional Properties

The last account of graded qualities in our partial taxonomy relies on what we will call *multidimensional properties*. Let us consider a paradigmatic example of a multidimensional property, the property of 'being wise'. Arguably, there

[15] Alternatively, one could treat them as two place relations in disguise, r_1 and r_2 now occupying the second argument place.

is not just one single way of being wise; rather, a person can be wise in very different ways.[16] To put it differently, wisdom has many different *aspects*. Think of e.g. being able to solve complex problems, being able to mediate between conflicting opinions, being well-informed about a wide range of subjects, and so on. Our third account says that in case an object x has a graded quality to a degree d_i, it has a multidimensional property P to that degree, where the value of d_i depends on, in a way to be discussed in further detail, how many aspects of P x has. To spell this idea out in a bit more detail, let us again fix the core principles which characterise the account:

Complexity and Aspects Multidimensional properties are complex properties consisting of combinations of aspect-properties.

Partly Having Objects partly have multi-dimensional properties by having their aspect-properties.

Degrees as Ratios The degree to which a multi-dimensional property is had by an object draws on the relation between the total number of aspect-properties of the multidimensional property and the number of aspect-properties which the object has.

We will not be able to provide a full account of multidimensional properties in this paper. The following discussion will hence involve a number of simplifications, including two concerning the complexity of the intrinsic structure of multidimensional properties.[17] The first is that we will restrict our focus to

[16] The commonly accepted top-level distinction between practical and theoretical wisdom already guarantees this. See Swartwood and Tiberius (2019), p. 19.

[17] The account we present here is similar to two existing accounts of complex properties, namely to an account of quantities introduced in Armstrong (1988) and to van Woudenberg and Peels's account of complex resultant properties (see van Woudenberg and Peels (2018)). A detailed comparison is beyond the scope of this paper. That being said, let us register a few similarities and dissimilarities, albeit briefly.

According to the first account, a quantitative property like 'having 2kg of mass' is constituted by a number of 'smaller' mass properties 'having 0.1kg of mass', 'having 1kg of mass', etc (see Eddon (2007), §I), just like a multidimensional property is constituted by its aspects. Armstrong assumes that this intrinsic structure can be used to explain comparative relations between amounts of mass, which parallels **Degrees as Ratios**. However, unlike in case of the multidimensional properties account, the difference between the constituting mass properties is merely quantitative and it does not (and cannot, since this would make it an inadequate account of quantities) accommodate the full range of distinctions between different modes of having we introduce in §4.

Van Woudenberg and Peels argue that certain true degree sentences imply that a thing has a specific kind of complex property P, a 'complex resultant property', which is composed of stereotypical, P-making properties and which is sensitive to a degree, even though it is not a quantitative property. Unlike Armstrong, they also allow qualitative variance between the (P-making) properties which constitute the complex properties they posit. Just like the Armstrongian account however, theirs does not accommodate different modes of having a property. Having a complex resultant property is still exclusively a binary matter, since such a property is had by an object if, and only if, the object has a subset of the constituting properties which is both necessary and sufficient for having the (complex) property (van

multidimensional properties which have a finite number of aspects. The second simplification directly connects to the first characteristic principle and will therefore be discussed in the following subsection.

Before we go on, we should point out that the multidimensional properties account and the other two might be seen as complementary, and not as exclusive alternatives. It might be better suited to account for particular graded qualities than the other two, and vice-versa. This suggests that we should take a pluralistic stance towards the generic phenomenon of metaphysical gradedness which we target in this paper and that different accounts of gradedness may be needed depending on the particular quality at issue.[18] The two examples discussed in detail in §5 give us a tentative argument in support of this view.

This view also suggests a second reason to deny that the multidimensional properties account and the other two are strict alternatives: **Complexity and Aspects** allows multidimensional properties to consist of aspect-properties which themselves correspond to graded qualities. According to the pluralist view, it might be the case that their gradedness is best accounted for in terms of one of the two other accounts, calling for an account of graded qualities which integrates the multidimensional and at least one of the other two accounts. Note that the following characterization is neutral regarding this issue and applies equally well if pluralism is correct, or if it is false so that the gradedness of aspects is accounted for by 'multidimensionality all the way down'.

3.5.1 Complexity and Aspects

We take the following to be definitional of multidimensional properties: if a property P is multidimensional, then it is constituted by a *proper plurality* of aspect properties $\mathbf{A} = P_1, ..., P_n$.[19] This re-statement of **Complexity and Aspects** makes it clear that the notion of constitution is of central importance when it comes to understanding what multidimensional properties are. The principle however leaves it open what exactly 'constituted' means in this context. While this is indeed a substantial metaphysical question, it is one which we can, for the purposes of this paper, leave more or less open. We simply need not address it to argue that multidimensional properties can provide us with an

Woudenberg and Peels (2018), §5). Again, our account specifically allows for different modes of having, including in particular 'having to a degree d' (see §4). This is not only a substantial difference between their account and one of our three accounts of what it means to have a quality to a degree, but it also shows that van Woudenberg and Peels are not engaged in the same general project we pursue in this paper.

[18] Note that van Woudenberg and Peels (2018) also advocate a pluralist stance, but their pluralism concerns the metaphysical implications of degree-involving sentences in general. They think that some such sentences do not imply the existence of metaphysical gradation, while others do, namely those referring to resultant complex properties. The pluralism we tentatively advocate in particular concerns *different accounts* of metaphysical gradedness.

[19] By 'proper plurality' we simply mean a plurality which has more than one member.

account of graded qualities.

A question we do need to address concerns the complexity arising from the compositional structure of multidimensional properties. Since aspects are themselves properties, it is natural to ask whether they can also correspond to graded qualities. Indeed, the examples of wisdom given above clearly suggests that they can.

A second natural question is whether the aspects of multidimensional properties can differ in how much they contribute to an object's having a multidimensional property of which they are constitutive. Based on these two question, one can in general distinguish a number of different types of multi-dimensional properties which (at least) differ regarding their complexity along two axes. The first is the 'weight' axis, which measures the overall contribution of a particular aspect to having the multidimensional property. The second a 'gradability' axis, which registers whether the aspects are themselves graded properties or not. Under the assumption that these two axes are logically independent we get four distinct types of multidimensional properties. Of these, we will here put aside all but the simplest type, multidimensional properties which have plain, that is non-multidimensional aspects, which all have the same weight, conforming to:

- **Same Weight, Plain Aspects** The multidimensional property P is constituted by n-aspect properties $\mathbf{A} = P_1, ..., P_n$, such that each P_i in $\mathbf{A}$ i) has the same weight and is ii) not a graded property.

This is the second simplification we will adopt in this paper. This of course means that the account offered here is not a full theory, but rather a proof of concept which is limited in scope to graded qualities which can be modelled using a particular type of multidimensional property.

3.5.2 Partly Having

We will further discuss this principle shortly, but we can already point out a crucial difference to the other two accounts. The third account provides a way of understanding 'partly talk' in terms of, broadly speaking, the compositional structure of the relevant multidimensional property. As of now, we simply want to note that, restating **Partly Having** in slightly more formal terms, for any quality $\mathcal{Q}$ which corresponds to a multidimensional property P, for some x to be partly $\mathcal{Q}$ is for x to have some aspect properties $P_1, ..., P_n$ which constitute P, but not all of them.

3.5.3 Degrees as Ratios

Suppose the multidimensional property P corresponding to a quality $\mathcal{Q}$ is constituted by n aspect properties $\mathbf{A} = P_1, ..., P_n$. Also suppose that x has exactly m of these aspect properties. For x to have $\mathcal{Q}$ it needs to have at least one aspect property P_i in $\mathbf{A}$. Since we are here solely focusing on multidimensional properties which conform to **Same Weight, Plain Aspects**, we can identify the degree to which x is $\mathcal{Q}$ with the ratio $\frac{m}{n}$. That is, we say that x has $\mathcal{Q}$ to degree $d = \frac{m}{n}$ if, and only if, x has exactly m of the total of n aspects of P.

As we have remarked earlier, the available values for degrees depend on the complexity of the multidimensional property, which means that they may fall short of being uncountably many. To have uncountably many degree-values, there have to be uncountably many aspects. It is worth mentioning a special case at the other end of the spectrum: A multidimensional property with only two aspects effectively only has two degrees of having, namely 0.5 and 1. The property is not had at all, if a thing has none of its two aspects.

It is worth pointing out that this illustrates that the intrinsic structure of multidimensional properties gives the third account metaphysical resources which the other two accounts, at least as we presented them, do not have. Further developments of these accounts may well have to introduce comparable resources to, among other things, provide a comparable account of how degrees of having come about. This is of course not to say that the multidimensional properties account is per se superior, as it is also preliminary in its own ways.

3.5.4 Multidimensional properties provide an account of graded qualities

Intermediate Degrees is easily accounted for: Things have a graded quality which corresponds to a multidimensional property P with n-aspect properties $\mathbf{A} = P_1, ..., P_n$ to different intermediate degrees by having different numbers j and m of aspect properties P_i in $\mathbf{A}$, with $j \neq m \neq n$.

As in case of the degreed properties account, Absolute Qualities is met since the account only entails that qualities corresponding to multidimensional properties are graded, but not that every quality corresponds to a multidimensional property. This highlights a potential limitation on the range of possible applications of the account: It cannot account for the gradedness of qualities to which, for principled reasons, multidimensional properties cannot correspond. We will discuss a potential case of this sort in §5.1.

That said, we should point out that the account allows us to easily define the notion of *fully* having a quality: For every graded quality $\mathcal{Q}$ which corresponds to a multidimensional property P, x *fully* has $\mathcal{Q}$, if x has all of P's aspects.

To discuss Degree Connections, we will again focus on our example of a degree connection between foolishness and wisdom. Assuming that an object x has a quality $\mathcal{Q}$ to degree 1 if x has all aspects of the corresponding property, one can again ensure that this condition is met by stipulation: Letting $\mathbf{A_W}$ and $\mathbf{A_F}$ denote the sets of aspects of the two properties, the following principle, stated again using semi-formal notation, does the job: $\forall x(\exists W_i \in \mathbf{A_W}(W_i x) \rightarrow \exists F_i \in \mathbf{A_F}(\neg F_i x))$. Admittedly, there is much more to say about this condition, especially if the assumed degree connection involve intermediate degrees,[20] but we will have to leave it at that.

Let us next consider Comparative Relations. In the simple cases we consider, x has a quality $\mathcal{Q}$ to a higher degree than y if, and only if, x has more of the aspect-properties of the property corresponding to $\mathcal{Q}$ than y. Note that there may be philosophical reasons to exempt certain qualities from this way of accounting for comparative relations, as we will see in §5.3.

Regarding Partiality, the account involves a novelty, namely that unlike the other two accounts, it allows one to recover 'partly' talk first, and degree talk second:

From multidimensional properties to 'partly'-talk: x partly has a quality $\mathcal{Q}$ if, and only if, it has some aspect-properties of the multidimensional property P which corresponds to $\mathcal{Q}$, but not all of them.

Multidimensional properties and degrees: x has a quality $\mathcal{Q}$ to degree $d = \frac{m}{n}$ if, and only if, x has exactly m of the n aspect properties of P, where P corresponds to $\mathcal{Q}$.

One can recover 'partly'-talk first in this manner, but one is not forced to do so. The following principles instead recover 'partly'-talk from multi-dimensional properties by passing through degrees first:

- A property P is multidimensional if, and only if, there is a proper plurality $\mathbf{A}_P$ of aspect properties which constitute P.
- A property P is n-multidimensional if, and only if, P is a multidimensional property constituted by exactly n-aspect-properties.
- x is P to degree $d = \frac{m}{n}$ if, and only if, x has exactly m aspect properties of the n-multidimensional property P.

[20] One difficulty in such cases is that one needs to ensure the comparability of degrees derived from multidimensional properties constituted by different numbers of aspects. Given our simplifying assumption about complexity, this is in principle just a matter of finding a common denominator, but things of course get more complicated once weights and graded aspects enter the picture.

- x partly has a quality $\mathcal{Q}$ if, and only if, x is P to a degree $d < 1$, where P corresponds to $\mathcal{Q}$.

The two ways to recover 'partly'-talk which we have just proposed are extensionally equivalent, so one may in principle freely chose between the two. Yet, there is a reason to prefer the first way. The sheer possibility of characterizing 'partly'-talk without first passing through degrees in the context of the multidimensional properties account clearly mark a significant difference to the other two accounts of graded properties. This difference is due to the compositional structure of multidimensional properties and the presence of this structure arguably gives one a reason to prefer the first way to account for graded qualities.

4 Fully and partly having, having simpliciter

The distinction between partly and fully having a quality has been in constant use throughout the paper. To repeat, an object *partly* has a quality if, and only if, it has it to a degree lesser than 1 and it *fully* has it if, and only if, it has it to degree 1.

Based on the preceding discussion of our three theories, we can introduce two further modes, or ways, in which graded qualities can be had. First, we can define a notion of 'weakly partly having,' which admits cases in which a thing fully has a quality:

Weakly partly having: x *weakly partly* has a quality $\mathcal{Q}$ if, and only if, it has $\mathcal{Q}$ to degree $d \leq 1$.

Weakly partly having relates to *partly having* similarly to how parthood relates to proper parthood, and likewise it relates to *fully having* in similar fashion as the existential relates to the universal quantifier.

In this context, one might ask: What about simply 'having a quality', or 'having a quality *simpliciter*'? In the end, even for allegedly graded qualities we do say things such as 'x is healthy,' or 'y is wise'.

Neither *partly*, *weakly partly*, nor *fully having* allow us to account for this. The point is best appreciated by focusing on multidimensional properties. Having a single wisdom aspect is sufficient for being partly and strictly partly wise, but is (given the mentioned simplifications) insufficient for being wise *simpliciter*. After all, a person who has only a single wisdom-aspect, e.g. that of being able to solve complex mathematical problems, but is foolish in a great number of ways, can hardly be called wise (simpliciter). The point is that 'having simpliciter' is more demanding than the two notions of partly having.

However, it is also less demanding than 'fully having'. Aristotle can arguably be considered to be a prototypical example of a wise person (i.e. he is *wise simpliciter*), but if the medieval cautionary tale of Phyllis and Aristotle is to be believed, even he was not fully wise. We want to propose one way of capturing these intuitions about the logical strength of 'having simpliciter': the notion can be defined with reference to what we shall call a *threshold* degree d_t, which is such that having a quality to d_t or a greater degree, is both necessary and sufficient for having the quality (simpliciter):

Having *simpliciter* (threshold definition): x has a quality $\mathcal{Q}$ *simplicter* if, and only if, x has $\mathcal{Q}$ to $d \geq d_t$.

It is natural to assume that the threshold degree d_t is sometimes *contextually fixed*. To give an example, the threshold degree for 'being a good organ player' in the context of a local church, and in the context of the entire history of music, including Bach, are evidently different.

That *having simpliciter* can depend on contextual factors, including cultural norms or perhaps even individual mental states, marks an important difference to the previous three notions of having, which are not sensitive to such factors. Still, the former is arguably the default target notion when we want to account for some ordinary uses of predicates. This also provides us with a response-strategy to an important objection:[21] Assume that there are two objects which have the same quality to only negligibly different degrees. Do the three accounts we propose commit one to the seemingly implausible claim that the metaphysical facts about these objects are so fine-grained that one of the two has, while the other lacks the quality?

The accounts do indeed imply that things have precise degrees (as we have not considered the possibility of metaphysically indeterminate gradation here), but the worry underlying the objection is arguably not aimed at this aspect of the accounts, but rather the modes or ways of having which we have defined for them. We take it that the most natural way to understand the objection is in terms of *having simpliciter*, i.e. as saying that whether something has a particular graded quality simpliciter may be overly sensitive to negligible differences in degree. (Note that the junctions between not having a graded quality at all (having it to degree 0) and having it *partly* and *fully* seem robust enough to avoid this sort of worry.)

One way to address the objection is to admit that there may be cases where we cannot determine the exact threshold degree which has to be met in order for a thing to have a quality simpliciter. E.g. in case of the multidimensional

[21] We thank one of our reviewers for this journal for raising this objection.

properties account, a single aspect may not be enough to turn a non-wise into a wise person. Importantly, one can coherently and plausibly accept this sort of indeterminacy, but still deny that it arises from the ontology of graded qualities postulated by the three accounts and instead locate the source of indeterminacy in our grasp of the contextual factors which crucially contribute to determining whether something has a quality simpliciter.

E.g. in case of wisdom, having simpliciter is plausibly sensitive to a potentially large number of historical and social factors that interact in complex ways, which may make it all but impossible for us to pinpoint the exact actual constellation of these factors, effectively preventing us from determining the exact context and corresponding threshold for having simpliciter. Accordingly, this response-strategy provides one with a notion of having simpliciter which conforms to the intuition that 'having a quality' should be tolerant with respect to neglible differences in degree, but keeps indeterminacy out of the metaphysical core-structure of the proposed accounts. More could be said, but we will have to leave it at that in this paper.

For a graded quality $\mathcal{Q}$, we can sum up the relations between the defined notions as follows:

Way of Having $\mathcal{Q}$	**Definition**
Weakly Partly	x *weakly partly* has $\mathcal{Q}$ iff, x has $\mathcal{Q}$ to degree d and $0 < d \leq 1$.
Partly	x *partly* has $\mathcal{Q}$ iff, x has $\mathcal{Q}$ to degree d and $0 < d < 1$.
Simpliciter	x has $\mathcal{Q}$ *simpliciter* iff, x has $\mathcal{Q}$ to $d \geq d_t$.
Fully	x *fully* has $\mathcal{Q}$ iff, x has $\mathcal{Q}$ to $d = 1$.

Table 1: Ways/Modes of 'having $\mathcal{Q}$'

Under certain natural assumptions, in particular that $0 < d_t \leq 1$, the logical relations between these ways of having a quality can be depicted as in Fig. 1 below, arrows indicating entailment:

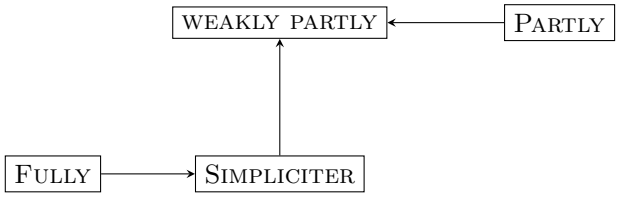

Figure 1: Ways/modes of 'having $\mathcal{Q}$': Logical Relations

This concludes our general investigation into graded qualities. In the rest of the paper we apply the results of this investigation to two concrete cases.

5 Application to Examples

In this section, we discuss two alleged examples of metaphysical gradedness, namely quantum observables, and value properties. The purpose of this discussion is to show that the choice between the three different accounts we introduced in §§3.1-3 matters, i.e. that this choice makes a real metaphysical difference.

One obvious difference concerns the applicability of the multidimensional properties account. As we will see, it is not obviously applicable in the first case. In the following, we argue that the degreed instantiation account is most apt in that case. We then go on to argue that the multidimensional properties account is most apt in case of the second example. To supplement our own arguments, we want to point to Nørgaard (2024), who argues that the gradation of a specific notion of quantum location is best accounted for using degreed properties. Note, again, that we do not mean to argue for the superiority of one of the three accounts of graded properties. Rather, our following discussion, plus the argument made in Nørgaard (2024), suggest a pluralist view, according to which different kinds of qualities may call for different metaphysical accounts of their gradedness.

5.1 Quantum Observables

Our first example of qualities which have been suggested to be graded are *quantum observables*. Without going into too much detail, in standard quantum mechanics, the complete description of a physical system is given by the so-called *quantum state*, represented by a vector $|\psi\rangle$ in a particular space called the Hilbert space of the system, $\mathcal{H}$. Observables, which we can, following our terminology, somewhat unconventionally refer to as quantum qualities, are represented by mathematical objects called Hermitian operators, $\hat{O}_i$. The value for a given operator $\hat{O}_i$ is represented by a real number. By way of illustration, the observable "spin$_x$", represented by $\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ has two possible values, namely $+\frac{1}{2}$ and $-\frac{1}{2}$. On some formulations, a physical system s in quantum state $|\psi\rangle$ has a determinate value v of a given $\hat{O}$ if, and only if, $\hat{O}|\psi\rangle = v|\psi\rangle$. In such a case, the state is said to be an *eigenfunction* of $\hat{O}$ corresponding to eigenvalue v.[22] In general, this will not hold for general states such as $|\psi\rangle = c_1|\psi\rangle_1 + c_2|\psi\rangle_2$, where c_1 and c_2 are $\neq 0$. Using spin x again as an example: $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$—where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenfunctions of $\hat{\sigma}_x$ corresponding to values $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively—is not an eigenfunction of $\hat{\sigma}_x$. In this case, some argue that 'spin$_x$' is graded. In particular, so the

[22] This is a version of the so-called *Eigenfunction-Eigenvalue Link*.

thought goes, the system s has '$\text{spin}_x = v$' to the degree $d = |c_i|^2$, where c_1 is the coefficient of the term of the eigenfunction corresponding to the relevant eigenvalue v. In the spin case, s has $\text{spin}_x = +\frac{1}{2}$ to degree $d = \frac{1}{2}$.[23]

Let us then see how the different accounts of graded qualities differ when applied to quantum observables. Here we take spin_x as a paradigmatic example, but it should not be difficult to generalise to other observables. We first note that, typically, quantum observables do not correspond to multidimensional properties. There simply are no natural candidates for properties which one could together take to be constitutive of spin_x.[24] Hence we will restrict our discussion to the *degreed properties* view and the *degreed instantiation* view.

According to the former, the *degreed-properties* view, there are strictly speaking uncountably many $+\frac{1}{2}$-spin properties, each having a particular degree d as a 'constituent'. In the notation we used in §3.2.1, these will constitute the set $\uparrow = \{+\frac{1}{2}_{d_i | i \in (0,1] \subset \mathbb{R}}\} = \{..., +\frac{1}{2}_{d\frac{1}{2}}, ..., +\frac{1}{2}_{d\frac{2}{3}}, ..., +\frac{1}{2}_{d_1}\}$. The same goes for $-\frac{1}{2}$-spin properties Indeed, in this case, it seems that these degrees are reflected in the coefficients of the particular quantum state the system happens to be in. As a way of illustration, consider the following quantum state namely $|\psi\rangle = (\sqrt{\frac{2}{3}}|\uparrow\rangle + \frac{1}{\sqrt{3}}|\downarrow\rangle)$. Then, the quantum system has the following quantum properties *simpliciter*: '$\text{spin}_x = +\frac{1}{2}_{d\frac{2}{3}}$' and '$\text{spin}_x = -\frac{1}{2}_{d\frac{1}{3}}$'.

According to the latter, the *degreed instantiation* view, there are only two maximally specific spin_x properties namely '$\text{spin}_x = +\frac{1}{2}$' and '$\text{spin}_x = -\frac{1}{2}$' but these are instantiated to different degrees in different quantum states. Reconsider the quantum state we mentioned above, $|\psi\rangle = (\sqrt{\frac{2}{3}}|\uparrow\rangle + \frac{1}{\sqrt{3}}|\downarrow\rangle)$. Then the relevant quantum system has the following quantum properties: '$\text{spin}_x = +\frac{1}{2}$' to degree $d = \frac{2}{3}$ and '$\text{spin}_x = -\frac{1}{2}$' to degree $d = \frac{1}{3}$.

It is clear that the two accounts provide very different metaphysical pictures. In the particular case at hand, this might provide reasons to tip the scale in favor of one account: it seems that the *degreed properties* account is (at least to some degree, no pun intended) more revisionary with respect to the usual formalism and its somewhat traditional interpretation. In any introductory class we are told that solving the eigenfunction-eigenvalue problem for $\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ yields only two eigenvectors, $|\chi^+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ and $|\chi^-\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

corresponding to only two eigenvalues $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively. Indeed,

[23] For further discussion of how degrees of having can be derived in quantum mechanics, see Nørgaard (2024).

[24] Perhaps one could approach the example from a different angle and argue that there is a way to apply the multidimensional properties account, since the property of *spin*, i.e. not spin along any particular axis, is multidimensional. Doing this would however require substantive arguments, which we cannot enter into here.

these are the only physically relevant quantum properties in this context. Not so, if the *degreed properties* view, which postulates an uncountable number of $+\frac{1}{2}$-spin-properties to account for the gradedness of spin_x, is correct. Now, according to the traditional interpretation of the formalism $\text{spin}_x = +\frac{1}{2}$ represents a *single property*. This traditional interpretation needs to be revised so as to yield that (somehow) it actually represents *uncountable many* physically relevant *properties*. Which property of the uncountably many gets instantiated at a given time would then depend on the quantum state the system is in. We are not claiming that this revision is intolerable. But it is a required revision nonetheless.

By contrast, the degrees of instantiation view does not require any revision of the usual formalism and its interpretation. Granted, it requires a revision of the somewhat orthodox metaphysical notion of instantiation. But since this is not a quantum mechanical notion, it seems that the degrees of instantiation account is less revisionary from a physical perspective. Perhaps this provides reasons enough to prefer it over the *degreed properties* view in the quantum case. That said, given that it is not our aim to defend a particular account—as we noted time and again—we will leave it at that.[25]

5.2 Value Properties

Values offer a further field of application for theories of graded qualities. Scruton for example takes it to be platitudinous that 'judgements of value tend to be comparative. When we judge things in respect of their goodness and beauty, our concern is very often to rank alternatives, with a view to choosing between them.' (Scruton (2009), p. 10.) These rankings rely on comparative relations, usually identified with the 'better than', 'worse than', 'equally good', and following an influential argument due to Chang (2002), also a fourth relation of being 'on a par.' These relations are taken to be relativised to a 'covering consideration,' (Chang (2002), p. 666) which specifies the respect in which the items are compared, in case of beauty, this might e.g. be the coherence of their composition, their elegance, etc.

Equipped with a theory of graded qualities, one can argue that the comparative nature of such value judgments has a direct metaphysical correspondence. Let us again consider the three accounts in turn and apply them to a salient example, a comparison between Virginia Woolf and Zaha Hadid regarding their creativity. According to the degreed-properties account, whether one of the two is more or less creative, as well as whether the two are equally creative or are on a par regarding their creativity, is a matter of the particular degreed creativity-

[25] Thanks to an anonymous referee for an insightful discussion.

properties they instantiate. Whether the judgment that Virginia Woolf is more creative than Zaha Hadid is correct, is accordingly a matter of the pair having the degreed 'creativity$_{d_i}$' and 'creativity$_{d_j}$' properties respectively, where i is strictly larger than j. According to the degreed-instantiation account, both would instead have to instantiate the same property, namely creativity, but they would have to do so by standing to it in the instantiation relation to a higher and a strictly lower degree respectively. The multi-dimensional properties account finally tells us that the judgement is correct just in case, assuming for the sake of illustration that creativity corresponds to the sort of simple multi-dimenstional property to which we limit our focus here, the ratio between the number of aspects of creativity which Virginia Woolf has and their total number is strictly larger than the same ratio for Zaha Hadid.

All three accounts arguably handle such comparisons well if we limit our attention to the first three comparative relations. If we follow Chang in making room for cases in which two items are on a par, the account based on multi-dimensional properties emerges as the most suitable. According to Chang, two items are on a par if they both have the value, but it is neither correct to judge either to be better or worse than the other, nor to judge them to be equally good with respect to it. The idea is that while both Virginia Woolf and Zaha Hadid were highly creative, the ways in which they manifested this quality were so different that it would neither be correct to judge them to be equally creative, nor to rank one of the two higher or lower with respect to their creativity. Neither the degreed-properties, nor the degreed-instantiation account seems to leave room for this fourth comparability-status—at least at first sight. Both the degreed-properties, as well as the degrees to which an object instantiates a base properties in a degree-property are strictly ordered and the two accounts offer little additional structure which could explain how two objects can both have a quality to a degree, but be only on a par. One might think that the problem could be solved by stipulating that Virginia Woolf and Zaha Hadid do not have degreed-properties involving the same base properties, or that they stand in the graded instantiation-relation to distinct creativity-properties. As a result, however, one could no longer maintain that both have the very same quality *simpliciter*, because there would be no family of degree-properties involving a unique base-property, and no unique property standing in the graded instantiation-relation to both of them, which would correspond to that quality. There may be other ways to account for parity in the context of the two first accounts, but such attempts could not mainly rely on the ontological structure they posit to account for gradedness.

If we think of creativity as corresponding to a multi-dimensional property, there is more room to maneuver. As in case of the other two accounts, one

can easily maintain that Virginia Woolf and Zaha Hadid both were creative simpliciter, because the ratio between the number of creativity-aspects which each of them had to the total number of aspects of creativity exceeds the relevant threshold. However, the multidimensional-properties account offers additional ontological structure which allows one to account for partity. The idea that the two are neither equally creative, nor such that one of the two is more, the other less creative could be implemented by introducing a requirement on the aptness of items to stand in the relations of betterness, worseness, or equality, which draw on the aspect-structure of the multidimensional property corresponding to the quality of being creative. The idea is to require a sufficient degree of overlap between those of its aspects which two items have in order for them to genuinely stand in either of the three relations with respect to each other. Accordingly, two items are on a par, if they each have a quality corresponding to a multidimensional property simpliciter, but lack the required overlap in aspects to be so comparable.

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